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Relativistic Hartree-Bogoliubov and QRPA description of exotic nuclear structure

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Abstract. The relativistic Hartree-Bogoliubov (RHB) model is extended to include density-dependent meson-nucleon vertex functions. Compared with standard relativistic mean-field effective interactions with non-linear meson-exchange terms, the density-dependent meson-nucleon couplings provide an improved description of asymmetric nuclear matter, neutron matter and nuclei far from stability. The relativistic quasiparticle random-phase approximation (RQRPA) is formulated in the canonical single-nucleon basis of the relativistic Hartree-Bogoliubov (RHB) model. Both in the particle-hole and particle-particle channels, the same interactions are used in the RHB calculation of the ground state and in the matrix equations of the RQRPA. The RHB+RQRPA is employed in the analysis of multipole excitations of neutron-rich oxygen isotopes.

PACS. 21.30.Fe Forces in hadronic systems and effective interactions – 21.60.Jz Hartree-Fock and random-phase approximations

1 Relativistic Hartree-Bogoliubov model with density-dependent meson-nucleon couplings

Quantum hadrodynamics (QHD) models, based on the mean-field approximation and with non-linear meson-exchange effective interactions, have been very successfully applied in the description of a variety of nuclear-structure phenomena, not only in nuclei along the valley of β-stability, but also in exotic nuclei with extreme isospin values and close to the particle drip lines. In particular, the relativistic Hartree-Bogoliubov (RHB) model, based on the relativistic mean-field theory and on the Hartree-Fock-Bogoliubov framework, provides a unified description of mean-field and pairing correlations. On the neutron-rich side, RHB studies include: the halo phenomenon in light nuclei [1], properties of light nuclei near the neutron-drip line [2], the reduction of the spin-orbit potential in nuclei with extreme isospin values [3], the deformation and shape coexistence phenomena that result from the suppression of the spherical $N = 28$ shell gap in neutron-rich nuclei [4], properties of neutron-rich Ni and Sn isotopes [5]. In proton-rich nuclei, the RHB model has been used to map the drip line from $Z = 31$ to $Z = 73$, and the phenomenon of ground-state proton radioactivity has been studied [6–8].

An alternative to QHD with non-linear effective interactions are models with density-dependent meson-nucleon vertex functions. Even though these two classes of models are essentially based on the same microscopic structure, *i.e.* on density-dependent interactions, the latter can be more directly related to the underlying microscopic nuclear interactions. In the framework of the density-dependent hadron field theory [9–11], the nucleons are described as point particles that move independently in the mean fields which originate from the nucleon-nucleon interaction. The theory is fully Lorentz invariant. Conditions of causality and Lorentz invariance impose that the interaction is mediated by the exchange of point-like effective mesons, which couple to the nucleons at local vertices. The single-nucleon dynamics is described by the Dirac equation

$$[\gamma^\mu(i\partial_\mu - \Sigma_\mu) - (m + g_\sigma\sigma)]\,\psi = 0\ , \qquad (1)$$

$$\Sigma_\mu = g_\omega\omega_\mu + g_\rho\tau\cdot\rho_\mu + e\frac{(1-\tau_3)}{2}A_\mu + \Sigma^R_\mu\ . \qquad (2)$$

σ, ω, and ρ are the meson fields, and A denotes the electromagnetic potential. The meson-nucleon couplings g_σ, g_ω, and g_ρ are assumed to be vertex functions of Lorentz-scalar bilinear forms of the nucleon operators. In most

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applications of the density-dependent hadron field theory the meson-nucleon couplings are functions of the vector density $\rho_{\mathrm{v}} = \sqrt{j_\mu j^\mu}$, with $j_\mu = \bar{\psi}\gamma_\mu\psi$. The density dependence of the vertex functions g_σ, g_ω, and g_ρ produces the *rearrangement* contribution Σ_μ^R to the vector self-energy,

$$\Sigma_\mu^R = \frac{j_\mu}{\rho_{\mathrm{v}}}\left(\frac{\partial g_\omega}{\partial \rho_{\mathrm{v}}}\bar{\psi}\gamma^\nu\psi\omega_\nu + \frac{\partial g_\rho}{\partial \rho_{\mathrm{v}}}\bar{\psi}\gamma^\nu\tau\psi\cdot\rho_\nu + \frac{\partial g_\sigma}{\partial \rho_{\mathrm{v}}}\bar{\psi}\psi\sigma\right). \tag{3}$$

The inclusion of the rearrangement self-energies is essential for the energy-momentum conservation and the thermodynamical consistency of the model [9].

In addition to the self-consistent mean-field potential, pairing correlations have to be included in order to describe ground-state properties of open-shell nuclei. In the framework of the relativistic Hartree-Bogoliubov model, the ground state of a nucleus is represented by the product of eigenvectors of the generalized single-nucleon Hamiltonian which contains two average potentials: the self-consistent mean-field $\hat{\Gamma}$ which encloses all the long-range particle-hole (ph) correlations, and a pairing field $\hat{\Delta}$ which sums up the particle-particle (pp) correlations. In the Hartree approximation for the self-consistent mean field, the relativistic Hartree-Bogoliubov equations read

$$\begin{pmatrix} \hat{h}_{\mathrm{D}} - m - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}_{\mathrm{D}} + m + \lambda \end{pmatrix}\begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}, \tag{4}$$

where $\hat{h}_{\mathrm{D}}$ is the single-nucleon Dirac Hamiltonian, and m is the nucleon mass. The chemical potential λ has to be determined by the particle number subsidiary condition in order that the expectation value of the particle number operator in the ground state equals the number of nucleons. $\hat{\Delta}$ is the pairing field. The column vectors denote the quasiparticle spinors and E_k are the quasiparticle energies. The RHB equations are solved self-consistently, with potentials determined in the mean-field approximation from solutions of Klein-Gordon equations for the meson fields. In most applications of the RHB model, pairing correlations have been described by the pairing part of the finite-range Gogny interaction.

The density dependence of the meson-nucleon couplings is parameterized [10],

$$g_i(\rho) = g_i(\rho_{\mathrm{sat}})f_i(x) \quad \text{for} \quad i = \sigma, \omega\ , \tag{5}$$

where

$$f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \tag{6}$$

is a function of $x = \rho/\rho_{\mathrm{sat}}$, and ρ_{sat} denotes the baryon density at saturation in symmetric nuclear matter. The five constraints $f_i(1) = 1$, $f_\sigma''(1) = f_\omega''(1)$, and $f_i''(0) = 0$, reduce the number of independent parameters to three. Three additional parameters in the isoscalar channel are $g_\sigma(\rho_{\mathrm{sat}})$, $g_\omega(\rho_{\mathrm{sat}})$, and m_σ —the mass of the phenomenological sigma-meson. For the ρ-meson coupling the functional form of the density dependence is suggested by Dirac-Brueckner calculations of asymmetric nuclear matter:

$$g_\rho(\rho) = g_\rho(\rho_{\mathrm{sat}})\ \exp\left[-a_\rho(x-1)\right]\ . \tag{7}$$

The isovector channel is parameterized by $g_\rho(\rho_{\mathrm{sat}})$ and a_ρ. For the masses of the ω and ρ mesons the free values are used: $m_\omega = 783$ MeV and $m_\rho = 763$ MeV. In ref. [11], we have introduced the density-dependent meson-exchange effective interaction (DD-ME1): the seven coupling parameters and the σ-meson mass have been simultaneously adjusted to properties of symmetric and asymmetric nuclear matter, and to ground-state properties (binding energies, charge radii and differences between neutron and proton radii) of twelve spherical nuclei.

Nuclear-matter properties calculated with the DD-ME1 interaction have been compared with those obtained with the density-dependent effective interaction TW-99 [10], and with two standard non-linear parameter sets NL3 [12] and NL1 [13]. The later non-linear effective interactions have been used extensively in studies of nuclear-structure phenomena over the whole periodic table, from light nuclei to superheavy elements. For symmetric nuclear matter all four interactions display similar saturation densities (with NL3 at the low end), and binding energies per nucleon (with NL1 at the high end). The principal difference between the density-dependent effective interactions DD-ME1 and TW-99 on the one hand, and the non-linear interactions NL3 and NL1 on the other, are the properties of asymmetric matter. The energy per particle of asymmetric nuclear matter can be expanded about the equilibrium density ρ_{sat} in a Taylor series in ρ and α:

$$E(\rho,\alpha) = E(\rho,0) + S_2(\rho)\alpha^2 + S_4(\rho)\alpha^4 + \ldots, \alpha \equiv \frac{N-Z}{N+Z}\,. \tag{8}$$

$$E(\rho,0) = -a_v + \frac{K_0}{18\rho_{\mathrm{sat}}^2}\ (\rho-\rho_{\mathrm{sat}})^2 + \ldots \tag{9}$$

and

$$S_2(\rho) = a_4 + \frac{p_0}{\rho_{\mathrm{sat}}^2}\ (\rho-\rho_{\mathrm{sat}}) + \frac{\Delta K_0}{18\rho_{\mathrm{sat}}^2}\ (\rho-\rho_{\mathrm{sat}})^2 + \ldots \tag{10}$$

The empirical value at saturation density $S_2(\rho_{\mathrm{sat}}) = a_4 = 30 \pm 4$ MeV. The parameter p_0 defines the linear density dependence of the symmetry energy, and ΔK_0 is the correction to the incompressibility. The non-linear effective interactions NL1 and NL3 have a considerably larger value a_4 of the symmetry energy at saturation density. This is also true for other standard non-linear parameter sets, and is due to the fact that the isovector channel of these effective forces is parameterized by a single constant, the density-independent ρ-meson coupling g_ρ. With a single parameter in the isovector channel it is not possible to reproduce simultaneously the empirical value of a_4 and the masses of $N \neq Z$ nuclei. This only becomes possible if a density dependence is included in the ρ-meson coupling, as is done in TW-99 and DD-ME1. In a recent study of neutron radii in non-relativistic and covariant mean-field models [14], the linear correlation between the neutron skin and the symmetry energy has been analyzed. In particular, the analysis has shown that there is a very strong linear correlation between the neutron skin thickness in ^{208}Pb and the individual parameters that determine the symmetry energy $S_2(\rho)$: a_4, p_0 and ΔK_0. The empirical

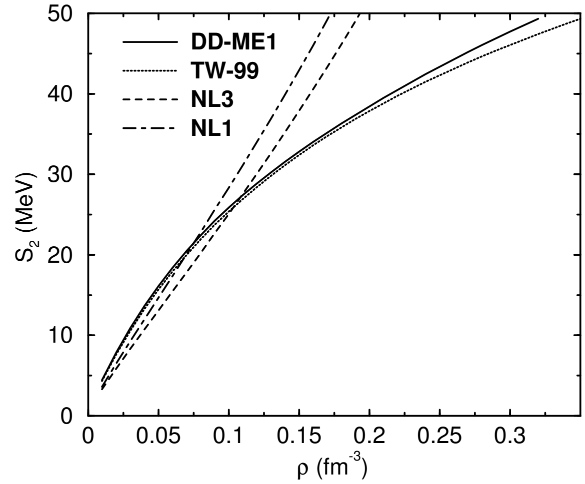

Fig. 1. The asymmetry energy as a function of the baryon density, calculated with the four relativistic interactions DD-ME1, TW-99 [10], NL3 [12], and NL1 [13].

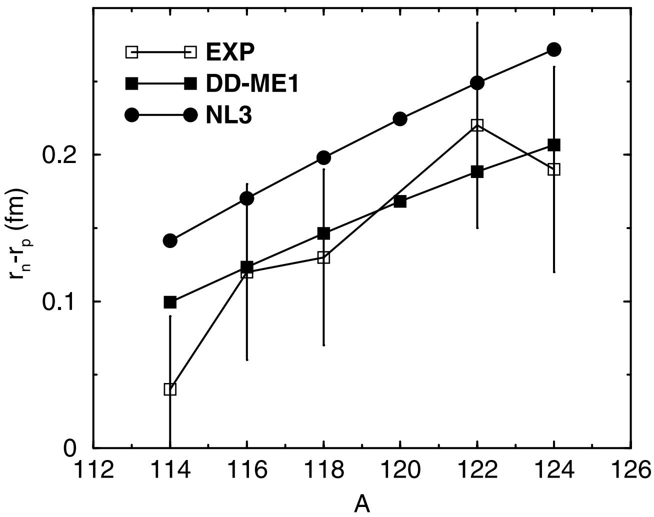

Fig. 2. DD-ME1 and NL3 predictions for the differences between neutron and proton rms radii of Sn isotopes, compared with experimental data from ref. [15].

value of $r_n - r_p$ in ^{208}Pb (0.20 ± 0.04 fm from proton scattering data) places the following constraints on the values of the parameters of the symmetry energy: $a_4 \approx 30$–34 MeV, 2 Mev/fm$^3 \leq p_0 \leq 4$ Mev/fm^3, and -200 MeV $\leq \Delta K_0 \leq -50$ MeV. While these constraints are satisfied by the density-dependent interactions DD-ME1 and TW-99, the parameters of the symmetry energy of the non-linear interactions are systematically much larger. In particular, p_0 is too large by a factor ≈ 2, and the correction to the incompressibility ΔK_0 has even a wrong sign for the two non-linear interactions. The qualitatively different density dependence of the symmetry energy for the two classes of effective interactions is illustrated in fig. 1, where we plot the coefficient S_2 as a function of the baryon density. Due to the very large value of p_0 and the small absolute value of ΔK_0, for NL3 and NL1, S_2 displays an almost linear density dependence of ρ. For the two density-dependent interactions, on the other hand, the quadratic term of S_2 dominates, especially at densities $\rho \geq 0.1$ fm^{-3}.

In ref. [11] the relativistic Hartree-Bogoliubov (RHB) model with the density-dependent interaction DD-ME1 in the *ph*-channel, and with the finite-range Gogny interaction D1S in the *pp*-channel, has been tested in the analysis of ground-state properties of the Sn and Pb isotopic chains. In fig. 2 we plot the calculated differences between radii of neutron and proton ground-state distributions of Sn nuclei. The non-linear interaction NL3 systematically predicts larger values of $r_n - r_p$. This effect is even more pronounced for the older parameter set NL1. The difference between the values calculated with NL3 and DD-ME1 increases with the number of neutrons to about 0.1 fm at $N = 82$, but then it remains practically constant for $N > 82$. The calculated values of $r_n - r_p$ are compared with experimental data [15]. While both interactions reproduce the isotopic trend of the experimental data, NL3 obviously overestimates the neutron skin. The values calculated with DD-ME1 are in excellent agreement with the experimental data, this result shows that the isovector channel of the effective interaction DD-ME1 is correctly parameterized.

The RHB model with density-dependent meson-nucleon couplings represents a significant improvement in the relativistic mean-field description of the nuclear many-body problem and, in particular, of exotic nuclei far from β-stability. The improved isovector properties of the effective interaction in the *ph*-channel on the one hand, and the unified description of mean-field and pairing correlations in the Hartree-Bogoliubov framework on the other, offer a unique possibility for accurate studies of nuclei with extreme ground-state isospin values and with Fermi levels close to the particle continuum.

2 QRPA based on the relativistic Hartree-Bogoliubov model

The multipole response of unstable nuclei far from the line of β-stability presents a very active field of research, both experimental and theoretical. These nuclei are characterized by unique structure properties: the weak binding of the outermost nucleons and the effects of the coupling between bound states and the particle continuum. On the neutron-rich side, in particular, the modification of the effective nuclear potential leads to the formation of nuclei with very diffuse neutron densities, to the occurrence of the neutron skin and halo structures. These phenomena will also affect the multipole response of unstable nuclei, in particular the electric dipole and quadrupole excitations, and new modes of excitations might arise in nuclei near the drip line.

A quantitative description of ground-states and properties of excited states in nuclei characterized by the closeness of the Fermi surface to the particle continuum, necessitates a unified description of mean-field and pairing correlations, as for example, in the framework of the

Hartree-Fock-Bogoliubov (HFB) theory. In order to describe transitions to low-lying excited states in weakly bound nuclei, in particular, the two-quasiparticle configuration space must include states with both nucleons in the discrete bound levels, states with one nucleon in the bound levels and one nucleon in the continuum, and also states with both nucleons in the continuum. This cannot be accomplished in the framework of the BCS approximation, since the BCS scheme does not provide a correct description of the scattering of nucleonic pairs from bound states to the positive energy particle continuum. Collective low-lying excited states in weakly bound nuclei are best described by the quasiparticle random-phase approximation (QRPA) based on the HFB theory.

The relativistic random-phase approximation (RRPA) has been recently employed in quantitative analyses of collective excitations in nuclei. Two points are essential for the successful application of the RRPA in the description of dynamical properties of finite nuclei: i) the use of effective Lagrangians with non-linear self-interaction terms, and ii) the fully consistent treatment of the Dirac sea of negative-energy states. The RRPA with non-linear meson interaction terms, and with a configuration space that includes the Dirac sea of negative-energy states, has been very successfully employed in studies of nuclear compressional modes [16,17], of multipole giant resonances and of low-lying collective states in spherical nuclei [18], of the evolution of the low-lying isovector dipole response in nuclei with a large neutron excess [19,20], and of toroidal dipole resonances [21].

In the relativistic framework, the QRPA can be formulated in the canonical single-nucleon basis of the relativistic Hartree-Bogoliubov (RHB) model. The eigensolutions of the RHB eq. (4) form a set of orthonormal single-quasiparticle states. The corresponding eigenvalues are the single-quasiparticle energies. Any RHB wave function can be written either in the quasiparticle basis as a product of independent quasiparticle states, or in the canonical basis as a highly correlated BCS state. The canonical basis is specified by the requirement that it diagonalizes the single-nucleon density matrix . The transformation to the canonical basis determines the energies and occupation probabilities of single-nucleon states, that correspond to the self-consistent solution for the ground state of a nucleus. Since it diagonalizes the density matrix, the canonical basis is localized. It describes both the bound states and the positive-energy single-particle continuum.

Taking into account the rotational invariance of the nuclear system, the matrix equations of the relativistic quasiparticle random-phase approximation (RQRPA) read

$$\begin{pmatrix} A^J & B^J \\ B^{*J} & A^{*J} \end{pmatrix} \begin{pmatrix} X^{\nu,JM} \\ Y^{\nu,JM} \end{pmatrix} = \omega_\nu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X^{\nu,JM} \\ Y^{\nu,JM} \end{pmatrix} . \tag{11}$$

For each RQRPA energy ω_ν, (X^ν) and (Y^ν) denote the corresponding forward- and backward-going two-quasiparticle amplitudes, respectively. The coupled RQRPA matrices in the canonical basis read

$$\begin{aligned} A^J_{kk'll'} = {} & H^{11(J)}_{kl}\delta_{k'l'} - H^{11(J)}_{k'l}\delta_{kl'} - H^{11(J)}_{kl'}\delta_{k'l} \\ & + H^{11(J)}_{k'l'}\delta_{kl} + \frac{1}{2}(\xi^+_{kk'}\xi^+_{ll'} + \xi^-_{kk'}\xi^-_{ll'})V^J_{kk'll'} \\ & + \zeta_{kk'll'}\tilde{V}^J_{kl'k'l} , \end{aligned} \tag{12}$$

$$\begin{aligned} B^J_{kk'll'} = {} & \frac{1}{2}(\xi^+_{kk'}\xi^+_{ll'} - \xi^-_{kk'}\xi^-_{ll'})V^J_{kk'll'} \\ & + \zeta_{kk'll'}(-1)^{j_l - j_{l'} + J}\tilde{V}^J_{klk'l'} . \end{aligned} \tag{13}$$

H^{11} denotes the one-quasiparticle terms,

$$H^{11}_{kl} = (u_k u_l - v_k v_l)h_{kl} - (u_k v_l + v_k u_l)\Delta_{kl} , \tag{14}$$

i.e. the canonical RHB basis does not diagonalize the Dirac single-nucleon mean-field Hamiltonian $\hat{h}_{\rm D}$ and the pairing field $\hat{\Delta}$. The occupation amplitudes v_k of the canonical states are eigenvalues of the density matrix. $\tilde{V}$ and V are the particle-hole and particle-particle residual interactions, respectively. Their matrix elements are multiplied by the pairing factors $\xi^\pm$ and ζ, defined by the occupation amplitudes of the canonical states. The relativistic particle-hole interaction $\tilde{V}$ is defined by the same effective Lagrangian density as the mean-field Dirac single-nucleon Hamiltonian $\hat{h}_{\rm D}$. $\tilde{V}$ includes the exchange of the isoscalar scalar σ-meson, the isoscalar vector ω-meson, the isovector vector ρ-meson, and the electromagnetic interaction. The two-body matrix elements include contributions from the spatial components of the vector fields:

$$\zeta_{kk'll'} = \begin{cases} \eta^+_{kk'}\eta^+_{ll'} & \text{for } \sigma,\ \omega^0,\ \rho^0,\ A^0 \text{ if } J \text{ is even}, \\ & \text{for } \boldsymbol{\omega},\ \boldsymbol{\rho},\ \boldsymbol{A} \text{ if } J \text{ is odd}, \\ \eta^-_{kk'}\eta^-_{ll'} & \text{for } \sigma,\ \omega^0,\ \rho^0,\ A^0 \text{ if } J \text{ is odd}, \\ & \text{for } \boldsymbol{\omega},\ \boldsymbol{\rho},\ \boldsymbol{A} \text{ if } J \text{ is even}, \end{cases}$$

with the η-coefficients defined by

$$\eta^\pm_{kk'} = u_k v_{k'} \pm v_k u_{k'} , \tag{15}$$

and

$$\xi^\pm_{kk'} = u_k u_{k'} \mp v_k v_{k'} . \tag{16}$$

The RQRPA configuration space includes the Dirac sea of negative-energy states. In addition to the configurations built from two-quasiparticle states of positive energy, the RQRPA configuration space must also contain pair-configurations formed from the fully or partially occupied states of positive energy and the empty negative-energy states from the Dirac sea. The inclusion of configurations built from occupied positive-energy states and empty negative-energy states is essential for current conservation and the decoupling of spurious states, as well as for a quantitative comparison with the experimental excitation energies of giant resonances [16,22].

The RQRPA model is fully self-consistent: the same interactions, both in the particle-hole and particle-particle channels, are used in the RHB equations that determines the canonical quasiparticle basis, and in the RQRPA equations. In both channels the same strength parameters of

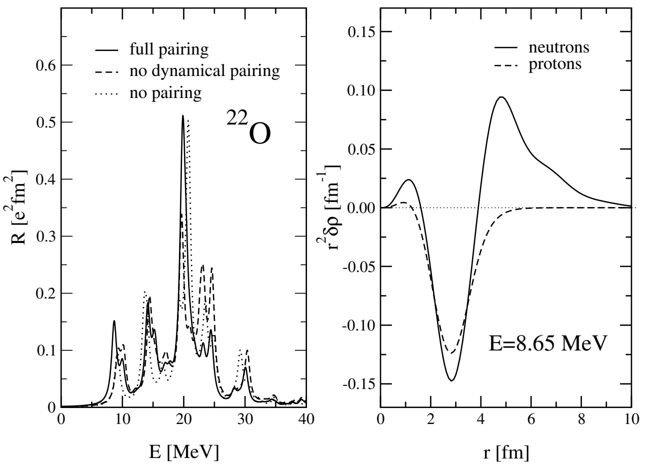

Fig. 3. The isovector dipole strength function for ^{22}O (left). The proton and neutron transition densities for the peak at $E = 8.65$ MeV are shown in the right panel.

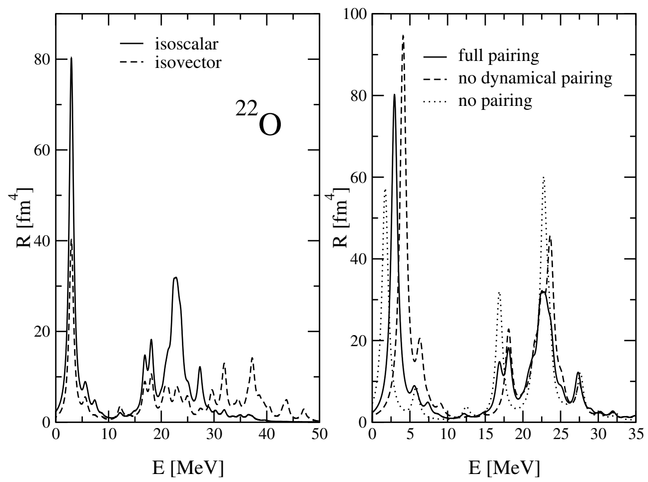

Fig. 4. The RHB+RQRPA isoscalar and isovector quadrupole strength distributions for ^{22}O (left panel). In the right panel the full RHB+RQRPA isoscalar strength function (solid) is compared to the RMF+RRPA calculation without pairing (dotted), and with the response obtained when the pairing interaction is included only in the RHB ground state (dashed).

the interactions are used in the RHB and RQRPA calculations. The parameters of the effective interactions are completely determined in RHB calculations of ground-state properties, and no additional adjustment is needed in RQRPA calculations. This is an essential feature and it ensures that RQRPA amplitudes do not contain spurious components associated with the mixing of the nucleon number in the RHB ground state (for 0^+ excitations), or with the center-of-mass translational motion (for 1^- excitations).

In order to illustrate the RHB+RQRPA approach we analyze the isovector dipole and isoscalar quadrupole response of ^{22}O. The calculation is fully self-consistent: the same combination of effective interactions, NL3 in the *ph*-channel and Gogny D1S in the *pp*-channel, are used both in the RHB calculation of the ground state and as RQRPA residual interactions. Similar calculations for the neutron-rich oxygen isotopes were recently performed in the framework of the non-relativistic continuum linear-response theory based on the Hartree-Fock-Bogoliubov formalism in coordinate state representation [23,24].

The isovector strength function ($J^\pi = 1^-$) of the dipole operator

$$\hat{Q}^{T=1}_{1\mu} = \frac{N}{N+Z}\sum_{p=1}^{Z} r_p Y_{1\mu} - \frac{Z}{N+Z}\sum_{n=1}^{N} r_n Y_{1\mu} \qquad (17)$$

for ^{22}O is displayed in the left panel of fig. 3. In this example we also compare the results of the RMF+RRPA calculations without pairing, with pairing correlations included only in the RHB ground state (no dynamical pairing), and with the fully self-consistent RHB+RQRPA response. A large configuration space enables the separation of the zero-energy mode that corresponds to the spurious center-of-mass motion. In the present calculation for ^{22}O this mode is found at $E = 0.04$ MeV.

The isovector dipole response in neutron-rich oxygen isotopes has recently attracted considerable interest because these nuclei might be good candidates for a possible identification of the low-lying collective soft mode (pygmy state), that corresponds to the oscillations of excess neutrons out of phase with the core composed of an equal number of protons and neutrons [25]. The strength functions shown in fig. 3 illustrate the importance of including pairing correlations in the calculation of the isovector dipole response. Pairing is, of course, particularly important for the low-lying strength below 10 MeV. The inclusion of pairing correlations in the full RHB+RQRPA calculation enhances the low-energy dipole strength near the threshold. For the main peak in the low-energy region (≈ 8.65 MeV), in the right panel of fig. 3 we display the proton and neutron transition densities. In contrast to the well-known radial dependence of the IVGDR transition densities (proton and neutron densities oscillate with opposite phases, the amplitude of the isovector transition density is much larger than that of the isoscalar component), the proton and neutron transition densities for the main low-energy peak are in phase in the nuclear interior, there is no contribution from the protons in the surface region, the isoscalar transition density dominates over the isovector one in the interior, and the strong neutron transition density displays a long tail in the radial coordinate. The effect of pairing correlations on the isovector dipole response in ^{22}O is very similar to the one obtained in the HFB+QRPA framework [24]. In the low-energy region below 10 MeV, however, the pairing interaction used in the QRPA calculation produces a much stronger enhancement of the dipole strength, as compared to the results shown in fig. 3. The reason probably lies in the choice of the pairing interaction. While we use the volume Gogny pairing, in ref. [24] a density-dependent delta force was used in the *pp* channel. This interaction is surface peaked and therefore produces a stronger effect on the low-energy dipole strength near the threshold. Nevertheless, we emphasize that the RHB+RQRPA results for the low-lying dipole

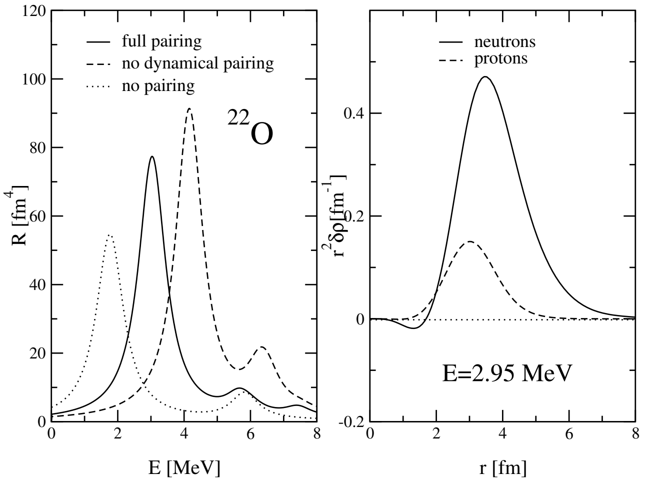

Fig. 5. Low-energy portion of the isoscalar quadrupole strength distribution in ^{22}O (left). The neutron and proton transition densities for the $J^{\pi} = 2^{+}$ state at $E = 2.95$ MeV (right).

strength distribution in ^{22}O are in very good agreement with recent experimental data [25].

In the left panel of fig. 4 we display the RHB+RQRPA isoscalar and isovector quadrupole ($J^{\pi} = 2^{+}$) strength distributions in ^{22}O. The low-lying $J^{\pi} = 2^{+}$ state is calculated at $E = 2.95$ MeV, and this value should be compared with the experimental excitation energy of the first 2^{+} state: 3.2 MeV [26]. The strong peak at $E = 22.3$ MeV in the isoscalar strength function corresponds to the isoscalar giant quadrupole resonance (IS GQR). The isovector response, on the other hand, is strongly fragmented over the large region of excitation energies $E \simeq 18$–38 MeV. The effect of pairing correlations on the isoscalar response is illustrated in the right panel of fig. 4, where again the full RHB+RQRPA strength function is compared to the RMF+RPA calculation without pairing, and with the response obtained when the pairing interaction is included only in the RHB ground state (no dynamical pairing). As one would expect, the effect of pairing correlations is not particularly pronounced in the giant resonance region. The inclusion of pairing correlations, however, has a relatively strong effect on the low-lying 2^{+} state. This is seen more clearly in the left panel of fig. 5, where only the low-energy portion of the isoscalar strength distributions in ^{22}O is shown. With respect to the RRPA calculation, the inclusion of the pairing interaction in the static solution for the ground state increases the excitation energy of the lowest 2^{+} state by ≈ 3 MeV. The fully self-consistent RHB+RQRPA calculation lowers the excitation energy from ≈ 4.5 MeV to $E = 2.95$ MeV. The inclusion of pairing correlations increases the collectivity of the low-lying 2^{+} state. The proton and neutron transition densities for the 2^{+} state at $E = 2.95$ MeV are shown in the right panel of fig. 5. They display a characteristic radial dependence. Both transition densities are peaked in the surface region, but the proton contribution is much smaller. The RQRPA results for the 2^{+} excitations are in agreement with non-relativistic QRPA calculations of the quadrupole response in neutron-rich oxygen isotopes [24,27].

The relativistic QRPA formulated in the canonical basis of the RHB model represents a significant contribution to the theoretical tools that can be employed in the description of the multipole response of unstable weakly bound nuclei far from stability.

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